ARBITRARY LAGRANGIAN-EULERIAN FORMULATION BASED ON THE COUPLED SPATIAL AND MATERIAL SETTING OF CONTINUUM MECHANICS

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The equilibrium equations of mechanics can be cast in two settings, namely the spatial setting (the direct motion problem) and the material setting (the inverse motion problem). The characteristic variables are the deformation gradient and the inverse deformation gradient, respectively. The direct motion problem concerns the equilibration of spatial forces, i.e. forces associated with a variation of the spatial coordinate while the material coordinate remains fixed. Conversely, in the inverse motion problem the material forces are equilibrated. Material forces are associated to a variation of the material coordinate while keeping the spatial coordinate fixed.

In a continuum sense, the direct and inverse motion problems are entirely equivalent. However, upon discretization discrepancies may arise: a vanishing residual of the spatial forces may coincide with a non-vanishing residual of the material forces. This phenomenon occurs when the nodal positions of the finite element mesh are sub-optimal. A release of energy takes place when a node is moved in the direction opposite to the accompanying material force residual. As such, a decrease of potential energy of the system is found.

In this contribution, we solve the direct motion problem and the inverse motion problem simultaneously, whereby the fundamental unknowns are not only the spatial coordinates but also the material coordinates of the nodes. Thus, an Arbitrary Lagrangian-Eulerian (ALE) formulation is obtained. In the proposed ALE method, the spatial forces and material forces are equilibrated simultaneously, so that finite element configurations are obtained that lead to the minimum potential energy possible. Therefore, the obtained meshes are optimal in the sense of potential energy.

In earlier ALE methods, the remeshing strategy (used to determine the material coordinates of the nodes) is normally taken as a more-or-less ad-hoc user-defined set of algebraic or differential equations, often including one or more user-defined remeshing parameters. On the other hand, in the proposed ALE scheme the remeshing strategy is an integral part of the equilibrium equations. Moreover, no additional parameters are needed.